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A Bayesian Analysis of the Deming Cost Model with Normally Distributed Sampling Data

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This article studies the Deming cost model using a Bayesian approach when the quality characteristic of items is assumed to have a normal distribution with unknown mean. Previously, researchers studied this model by the go/no-go data. Through a Bayesian approach, the model consists of a two-stage decision that minimizes the expected total cost: The first stage decision is to determine the optimal sample size, and the second stage decision is to decide whether to stop inspection or continue to inspect the remaining items of the lot. Numerical integration is used to find an approximate solution to the model. An illustrative example is given and a numerical analysis of this example is performed to realize the effects of the model parameters. The cost difference between using the measurement data and the corresponding go/no-go data under the same probability assumptions and cost structure is also investigated.

Keywords Bayesian decision analysis; Decision trees; Inspection sampling.

1. INTRODUCTION

Inspection procedure is often used as a tool for quality assurance in many manufacturing systems. If we are not sure the components in need are high quality or the quality of the process declines, then some procedure should be taken. In such situations, acceptance sampling plans and 100% inspection plans are common short-term approaches. There are various approaches in the determination of an inspection procedure, and the decision theoretic approach is probably the most reasonable method to model this problem on the basis of economic considerations and sampling information (Fink and Margavio, 1994).

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To classify an item in the lot as either defective or nondefective (go/no-go) is an attribute sampling inspection problem. To measure the quality of an item in the lot by a continuous scale is a variable sampling inspection problem. A variable sampling inspection problem can become an attribute sampling problem if the procedure only counts the number of items nonconforming to specification limit(s) in the sample and uses this number to decide whether the remaining items of the lot are accepted. To design a variable sampling plan, we need to specify the sample size and the acceptance limit(s). If the measured value from the sampling variables falls within the acceptance limit(s), the lot is accepted. Otherwise, the lot is rejected. Usually, the acceptance limit(s) will depend on the probability assumptions of the inspection model, the sample size, and the loss function. A step loss function implies that the customers are completely satisfied with the items conforming to the specifications and become completely unsatisfied when the value of the performance variable falls outside the specifications. A loss function is polynomial if the loss is polynomial in the value deviated from an ideal (target) value for an accepted item.

Moskowitz and Tang (1992) used the cost structure proposed by Schmidt et al. (1974) to develop a Bayesian variables acceptance sampling model with the following probability assumptions: The performance variable has a normal distribution with unknown mean, which is assumed to be normally distributed as well. Tagaras (1994) studied a similar cost structure under the same probability assumptions but assumed that the inspection was destructive and therefore the cost of inspection per unit was greater than the cost of rejection per unit. Yeh and Van (1997) developed a Bayesian double-variable sampling model with the polynomial loss function under the same probability assumptions.

Deming (1982) discussed a (n, c) rectifying attributes sampling plan relative to two different cost setups, (k_1, k_2), where n is the sample size, c is the acceptance number, k_1 is the cost per unit to inspect an item, and k_2 is the cost per unit of a nonconforming item that is either placed in an assembly that fails or that enters the stream of commerce and subsequently fails. Usually, the k_2 cost is much higher than the k_1 inspection cost. By rectifying inspection, we mean a procedure whereby a lot rejected by sampling inspection is 100% inspected and all nonconforming items discovered during inspection are replaced with conforming units. After observing the sampling outcome, if the number of nonconforming items in the sample of size n is greater than the acceptance number c, the lot is rejected and subject to 100% inspection. Otherwise, the lot is accepted and all remaining items of the lot are sent to assembly without inspection. It is noted that any nonconforming item found in the inspection stage and in the assembly stage will be replaced with a perfect item. The perfect item is obtained by inspecting items of the same quality from other resource. Depending on the contract, the extra inspection cost to obtain a perfect item can be charged to either the manufacturer or the supplier. An application example of the model is the manufacture of implantable prosthetic medical devices (Kaminsky and Haberle, 1995). Another example occurs in applying the surface mount technology (SMT) to printed circuit board (PCB) assembly, where the rework cost for replacing a non conforming electronic device mounted onto the board is cheaper than the scrap cost of the board.

Papadakis (1985), Burke et al. (1993), Vander Wiel and Vardeman (1994), and Kaminsky and Haberle (1995) discussed the Deming cost model by attributes from a classical statistics point of view (i.e., the fraction of conforming items in the lot, p, is known). Rigdon (1995) discussed in detail the case where p is unknown. In such situations, one might consider using a Bayesian approach to minimize total cost. Applying this approach will necessitate the need to describe p with a probability or density function based on knowledge and previous data. The book by Berger (1985) provides several methods for quantifying prior information as a distribution.

Lorenzen (1985) and Barlow and Zhang (1986) discussed the Deming cost model by attributes from a Bayesian point of view and provided computer codes for the model using beta prior for the probability of an item being conforming. In this article, we extend the Bayesian approach study of this model to the variable sampling plan, where the quality characteristic of items has a normal distribution with unknown mean but known standard deviation. An item is conforming if the value of its performance variable falls within a two-sided specification interval. Meanwhile, we are also interested in learning how much cost can be saved for this model when the variable measurement data is used as compared with the derived go/no-go type data.

The remainder of this article is organized as follows. In Section 2 the problem is described and a variable sampling model is formulated based on the Bayesian decision rule. Section 3 describes how to obtain the corresponding attributes sampling model from the variables sampling model. Section 4 illustrates an application example. Section 5 presents a numerical analysis of this model. Finally, the conclusions are summarized in the last section of this article.

2. VARIABLES SAMPLING MODEL

The Deming cost model uses rectifying inspection. A lot rejected by sampling inspection is subject to 100% inspected. It is also assumed that this inspection is 100% effective and that all nonconforming items discovered during inspection are replaced with conforming units. If a nonconforming item is put into the assembly and results in a bad product, the product can be repaired by the replacement of the nonconforming item. Any nonconforming item detected in the stages of inspection and assembly will be replaced with a perfect item. The perfect item is obtained by extra inspections on items of the same quality from other resources.

The decision process of the Deming cost model consists of two stages: (1) D_1: determine the sample size n, and (2) D_2: after observing the sampling outcome, decide whether to stop inspection at size n (denoted "Sn") or continue to inspect the N items of the lot (denoted "CN"). The model can be represented as the decision tree shown in Figure 1.

The notation and definitions of parameters or variables used are as follows:

N: Lot size
n: Sample size
X_j: Performance measure of the jth item in the lot
$\underline{X}_n$: Random vector $(X_1, X_2, \ldots, X_n)$
U: Mean value of performance variable, an unknown parameter
τ: Prior mean of U
γ: Standard deviation of the prior distribution of U
τ': Posterior mean of U
γ': Standard deviation of the posterior distribution of U
Z_n: Number of "nonconforming" items in the samples
y: Realization of Z_n

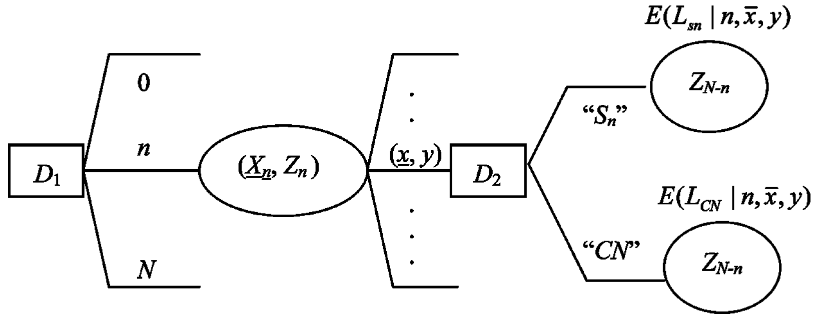

Figure 1. Decision tree of the problem.

Z_{N-n}: Number of "nonconforming" items in the remainder of the lot
$\overline{x}$: Mean of measured values from the samples
L_{Sn}: Total cost due to making decision "Sn" on the remainder of the lot
L_{CN}: Total cost due to making decision "CN" on the remainder of the lot
$M(y)$: Number of additional inspections to find y conforming items
$M(Z_{N-n})$: Number of additional inspections to find Z_{N-n} nonconforming items
$P(U)$: Probability of an item being conforming, a function of U

In this research, like most studies in variables sampling plans, we assume that the performance variables have conditional normal distributions; that is, $X_1, \ldots, X_N | U = u \sim i.i.d. N(u, \sigma^2)$. The prior of the unknown mean is also normal, $U \sim N(\tau, \gamma^2)$. The conditional distribution for the sample mean is normally distributed, $\overline{X}|U = u \sim N(u, \sigma^2/n)$. The unconditional distribution for $\overline{X}$ is also normal, which can be shown as follows:

$$\overline{X} - U \sim N(0, \sigma^2/n) \text{ is independent of } U \text{ and}$$
$$\overline{X} = \overline{X} - U + U \sim N(\tau, \gamma^2 + \sigma^2/n).$$

Because the number of nonconforming items, y, is a function of $\underline{x} = (x_1, x_2, \ldots, x_n)$, we have $f(u|x_1, x_2, \ldots, x_n, y) = f(u|x_1, x_2, \ldots, x_n) = f(u|n, \overline{x})$. By Bayes' theorem, the posterior of U given data $(x_1, x_2, \ldots, x_n)$ is as follows:

$$f(u|x_1, x_2, \ldots, x_n) \propto \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x_i - u}{\sigma}\right)^2} \cdot \frac{1}{\sqrt{2\pi}\gamma} e^{-\frac{1}{2}\left(\frac{u-\tau}{\gamma}\right)^2} \propto e^{-\frac{1}{2}\left(\frac{u-\tau'}{\gamma'}\right)^2},$$

where $\tau' = \left((\sigma^2\tau + n\gamma^2\overline{x})/(\sigma^2 + n\gamma^2)\right)$ and $\gamma' = (n/\sigma^2 + 1/\gamma^2)^{-1/2}$. Therefore, the posterior of U is again normally distributed with mean τ' and standard deviation γ', and sufficient statistics is $(n, \overline{x})$ because the posterior of U depends on the data, $(x_1, x_2, \ldots, x_n)$, only through the sample mean $\overline{x}$.

In the case of double specification limits, $[a, b]$, the probability of an item being conforming given $U = u$, $P(u) = \Pr\{a \le X_j \le b | U = u\} = \Phi((b - \mu)/\sigma) - \Phi((a - \mu)/\sigma)$ and the distribution of Z_n given $U = u$ is binomial (n, $P(u)$), where $\Phi(\cdot)$ represents the probability of the standard normal distribution.

Although the sampling outcome is $(n, \overline{x}, y)$, the total costs for decisions "Sn" and "CN", respectively, are as follows:

$$(L_{Sn}|n, \overline{x}, y) = n \cdot k_1 + M(y) \cdot k_1 + Z_{N-n} \cdot k_2 + M(Z_{N-n}) \cdot k_1 \tag{1}$$

$$(L_{CN}|n, \overline{x}, y) = n \cdot k_1 + M(y) \cdot k_1 + (N - n) \cdot k_1 + M(Z_{N-n}) \cdot k_1 \tag{2}$$

By taking the expected total cost as the comparison criterion, decision "Sn" is superior to decision "CN" if $E(L_{Sn}|n, \overline{x}, y) \le E(L_{CN}|n, \overline{x}, y)$. Note that this conclusion remains valid if the manufacturer is free of the extra inspection cost because both terms (1) and (2) contain $M(y) \cdot k_1 + M(Z_{N-n}) \cdot k_1$. After some algebraic operations (see Appendix), we obtain

$$E(L_{Sn}|n, \overline{x}, y) \le E(L_{CN}|n, \overline{x}, y) \quad \text{if and only if}$$
$$1 - E(P(U)|n, \overline{x}) \le \frac{k_1}{k_2} \tag{3}$$

where $E(P(U)|n,\overline{x}) = \int_{-\infty}^{\infty}\left[\int_a^b (1/\sqrt{2\pi}\sigma) \cdot e^{-\frac{1}{2}(x-u)^2 \cdot \sigma^{-2}} dx\right] \cdot f(u|n,\overline{x})du = \Pr\{a \le X_{n+1} \le b|n,\overline{x}\}$.

The distribution of $X_{n+1}|n, \overline{x}$ is $N(\tau', \sigma^2 + (\gamma')^2)$ because $X_{n+1}|n, \overline{x} = X_{n+1} - U + U|n, \overline{x}$ and $X_{n+1} - U|n, \overline{x} \sim N(0, \sigma^2)$ is independent of the data $(n, \overline{x})$ and the random variable $U|n, \overline{x} \sim N(\tau', (\gamma')^2)$. Thus, we have $E(P(U)|n, \overline{x}) = \Phi\left((b - \tau')/\left(\sqrt{\sigma^2 + (\gamma')^2 t}\right)\right)$

$-\Phi\left((a-\tau')/\left(\sqrt{\sigma^2+(\gamma')^2}\right)\right)$. Furthermore, because $\tau' = \left((\sigma^2\tau + n\gamma^2\bar{x})/(\sigma^2+n\gamma^2)\right)$ and γ' is independent of $\bar{x}$, $E(P(U)|n,\bar{x})$ reaches maximum when $\tau' = (a+b)/(2)$, which gives $\bar{x}^* = ((a+b)(\sigma^2+n\gamma^2) - 2\sigma^2\tau)/(2n\gamma^2)$. The value of $E(P(U)|n,\bar{x})$ goes down as τ' moves further away from $(a+b)/2$. The interval, $[\bar{x}_L, \bar{x}_R]$, for choosing decision "S_n" under the sample size of n can be computed as follows:

$$\bar{x}_L = \min\left\{\bar{x} : E(P(U)|n,\bar{x}) \geq 1 - \frac{k_1}{k_2},\ \bar{x} \leq \bar{x}^*\right\} \quad \text{and}$$
$$\bar{x}_R = \max\left\{\bar{x} : E(P(U)|n,\bar{x}) \geq 1 - \frac{k_1}{k_2},\ \bar{x} \geq \bar{x}^*\right\}. \tag{4}$$

In other words,

$$1 - E(P(U)|n,\bar{x}) \leq \frac{k_1}{k_2} \quad \text{if and only if } \bar{x} \in [\bar{x}_L, \bar{x}_R]. \tag{5}$$

The objective function of the model can be obtained by applying the Bayesian decision rule to the decision tree given in Figure 1.

$$\underset{0\leq n\leq N}{\mathrm{Min}} \int_{-\infty}^{\infty} \left[\sum_{y=0}^{n}\{\mathrm{Min}[E(L_{Sn}|n,\bar{x},y), E(L_{CN}|n,\bar{x},y)]\} \cdot \Pr\{y|n,\bar{x}\}\right] \cdot g(\bar{x}|n)d\bar{x} \tag{6}$$

where $g(\bar{x}|n)$ is the *p.d.f.* of the sufficient statistic $\bar{x}$. We want to determine a sample size n^* between 0 and N that finds the minimum expected total cost (METC) with respect to Eq. (6). This sample size n^* is referred to as the optimal sample size (OSS). Equation (6) can be simplified further as follows:

$$\underset{0\leq n\leq N}{\mathrm{Min}}\left\{n\cdot k_1 + (N-n)\cdot\left[E\left(\frac{1}{P(U)}\right)-1\right]\cdot k_1 + (N-n) \cdot \int_{-\infty}^{\infty}\mathrm{Min}[(1-E(P(U)|n,\bar{x}))\cdot k_2, k_1]\cdot g(\bar{x}|n)\cdot d\bar{x} + n\cdot k_1 \cdot \int_{-\infty}^{\infty} E\left(\frac{1}{P(U)}|n,\bar{x}\right)\cdot(1-E(P(U)|n,\bar{x})) \cdot g(\bar{x}|n)\cdot d\bar{x}\right\} \tag{7}$$

Detailed calculations related to obtaining Eq. (7) are given in the Appendix. In the case that the manufacturer is free of the extra inspection cost, Eq. (7) does not contain the second and the fourth terms.

The computation for Eq. (7) determines the OSS. After observing the sampling outcome $(x_1,\ldots,x_{oss})$, we compute $\bar{x}_{oss} = (x_1+\cdots+x_{oss})/OSS$. If $1-E(P(U)|OSS,\bar{x}_{oss}) \leq k_1/k_2$ or $\bar{x}_{oss} \in [\bar{x}_L, \bar{x}_R]$, we should choose $D_2 =$ "S_n." Otherwise, we choose $D_2 =$ "CN."

There is no closed form for Eq. (7) and Simpson's three-eighths integration approximation rule is applied to compute the integrals in this formula. During the computations, an accurate numerical table for standard normal distribution is helpful in computational efficiency because the posterior distribution of parameter U and the marginal distribution of sample mean $\overline{X}$ have normal distributions. Suppose that $(\bar{x}_1,\ldots,\bar{x}_M)$ are used to represent the values of $\overline{X}$ in the numerical integration. The value of the third term of Eq. (7) for a sample size of n can be computed as follows:

$Sum3 = 0$; {Begin}
For $j = 1$ to M
  If $\bar{x}_j \in [\bar{x}_L, \bar{x}_R]$, compute $(1 - E(P(U)|n,\bar{x}_j)\cdot k_2$ and add it to $Sum3$
  Else $Sum3 = Sum3 + k_1$;
$Sum3 = (N-n)\cdot Sum3$. {End}

Our computational experience shows that the value of Eq. (7) as a function of sample size n behaves like Figure 2, but the curve may not be smooth locally. Binary search for finding the METC and its corresponding OSS is not suitable to apply in such a situation. To reduce computational time and avoid missing the METC and its corresponding OSS, we adopt the following searching strategy.

Start with $n = 0$. Compute the values of Eq. (7) for sample sizes with multiples of 10. If the value goes up for three consecutive searches or to the lot size N, then the algorithm stops searching further and takes the sample size with minimum value up to the present search. If the sample size with minimum value is $10\cdot k$, then the algorithm searches the OSS within $[10\cdot(k-1), 10\cdot(k+1)]$ for $1 \leq k < N$, $[0, 10\cdot(k+1)]$ for $k = 0$, and $[N-10, N]$ for $k = N$.

3. ATTRIBUTES SAMPLING MODEL

In this section, we discuss how to establish an attributes sampling model based on the variable measurement data. In other words, the model will use the information of the number of nonconforming items that comes from the variable measurement data in the sample. This attributes sampling model needs the probability of a component being conforming, $P(U)$, which is a function of parameter U and is a random variable of the following form:

$$P(U) = \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-U}{\sigma}\right)^2} dx$$

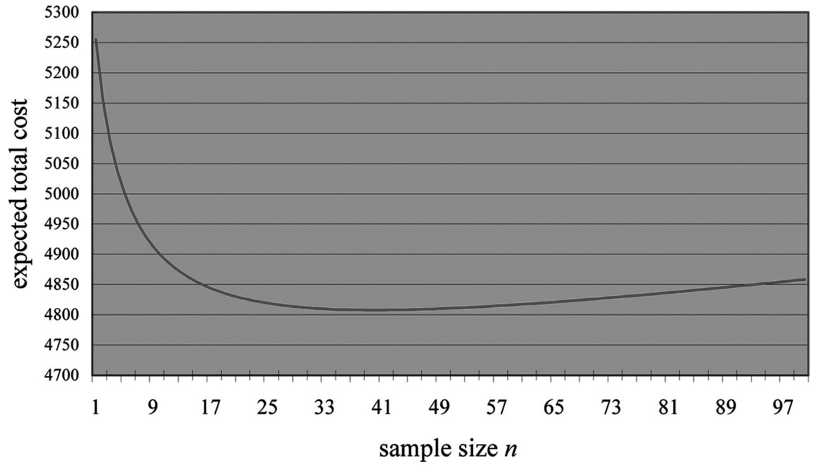

Figure 2. Expected cost for supplier B in sample size n.

There is no closed form for the continuous distribution of $P(U)$. The following method computes an approximate discrete probability distribution of $P(U)$.

The conditional probability under $U = u$, $P(u) = \Phi((b-u)/\sigma) - \Phi((a-u)/\sigma)$, is maximum when $u = (a+b)/2$ and decreases as u moves away from $(a+b)/2$. Partition the interval, [0,1], into M subintervals of equal length with $p_m = m/M$, $m = 1, 2, \ldots, M$. Let $u_m^L = \mathrm{Min}\{u : P(u) \geq m/M\}$ and $u_m^U = \mathrm{Max}\{u : P(u) \geq m/M\}$. We have $\Pr\{P(U) \geq p_m\} = \Phi\big((u_m^U - \tau)/\gamma\big) - \Phi\big((u_m^L - \tau)/\gamma\big)$ and $\Pr\{P(U) = p_m\} = \Pr\{P(U) \geq p_{m-1}\} - \Pr\{P(U) \geq p_m\}$.

After obtaining the approximation discrete distribution for $P(U)$, we observe the sampling data ($x_1, x_2, \ldots, x_n$) and compute the number of nonconforming units in the sample, $y = \sum_{k=1}^{n} y_k$, where $y_k = 0$ if $a \leq x_k \leq b$ and $y_k = 1$ otherwise.

In the attributes sampling model, (n, y) are sufficient statistics, and the probability distribution of $Y|n$, p_m is binomial.

Using the decision analysis similar to the variables sampling model, we conclude the following statements:

(S1) Model objective is $\mathrm{Min}_{0\leq n\leq N} \sum_{0\leq y\leq n} [\mathrm{Min}(E(L_{Sn}|n,y), E(L_{CN}|n,y))] \cdot \Pr\{y|n\}$.
(S2) Decision "Sn" is superior to "CN" if and only if $(1 - E(P(U)|n,y)) \leq \frac{k_1}{k_2}$.

Barlow and Zhang (1986) showed that the posterior distribution of $P(U)$ has the following property: For any prior distribution of $P(U)$, $E(P(U)|n, y)$ increases in n (y fixed) and $E(P(U)|n, y)$ decreases in y (n fixed). We can use this property to facilitate the computations for the attributes sampling model.

By (S2) and the property stated preiously, if $1 - E(P(U)|n,0) > k_1/k_2$, then decision "$CN$" is better regardless of the sampling outcome. Likewise, by (S2) and the property, if $1 - E(P(U)|n,n) \leq k_1/k_2$, zero inspection is better than taking sample size n because any sampling outcome in this sample size will not change the decision "Sn." For the other case, there exists an acceptance number, c, $0 \leq c < n$, satisfying $1 - E(P(U)|n,c) \leq k_1/k_2$ and $1 - E(P(U)|n, c+1) > k_1/k_2$. We would choose "$Sn$" if $y \leq c$ and choose "CN" otherwise.

4. AN EXAMPLE

A producer decides to run a pilot batch of $N = 500$ articles, say, notebooks of a new type. Along with the new product is a key component, a power adaptor, which must provide strictly stable DC voltage at a specification of $(a, b) = (23.95, 24.05)$ V. The producer itself does not manufacture the power adaptors and must purchase them from outside resource. The inspection cost for power adaptor per item is $k_1 = \$9.25$, and the product failure cost (a product delivered to customer with a bad power adaptor) of k_2 is estimated at \$72.40 each. The product failure cost includes transportation cost, customers' dissatisfaction, and so on. The producer cooperates with three supplier candidates (A, B, and C) and owns the data of each supplier's adaptor during the R&D period. Assume that the purchase cost per item is the same for all three suppliers. Based on the technologies and data, the producer decides to take the values of three parameters, (σ, τ, γ), with respect to the three suppliers as shown in Table 1. The value of τ

Table 1
METC and OSS for suppliers A, B, and C

Supplier	σ	τ	γ	*OSS*	METC
A	0.0231	24.0241	0.00962	42	$4,835
B	0.0282	24.0137	0.01260	40	$4,807
C	0.0235	24.0249	0.01270	37	$5,063

could refer to the total mean performance measurement of components in previous lots, and γ could refer to the lot-by-lot variability of the mean performance measurement. Assume that the producer bears the regular and extra inspection costs. The computational results show that the best supplier is B with METC at $4,807 and an optimal sample size $OSS = 40$. In addition, by Eq. (4), we obtain $[\bar{x}_L, \bar{x}_R] = [23.9801,\ 24.0165]$.

Assuming that after inspecting the samples of 40 components from supplier B, the sample mean and the number of nonconforming items are $\bar{x} = 23.985$ and $y = 7$, respectively. Because $\bar{x} = 23.985$ is within the interval $[\bar{x}_L, \bar{x}_R]$, we should stop inspection and send the remaining components of the lot into assembly. Another computation results in the two expected total costs $E(L_{Sn}|OSS, \bar{x}) = \$4,675$ and $E(L_{CN}\ |OSS, \bar{x}) = \$5,424$, respectively. The expected number of extra inspections for compensation is $E(M(y)|OSS,\ \bar{x}) + E(M(Z_{N-n})|OSS,\ \bar{x}) = 86.3$. All three values are obtained by applying the Simpson's three-eighths approximation integration rule to the formulae given in the Appendix. The expected extra inspection ratio at the second stage decision is 0.1765, which is obtained by dividing the expected number of extra inspections by the lot size $N = 500$. The producer will pay the extra inspection cost but no extra purchase cost.

5. NUMERICAL ANALYSIS

In this section, we present a numerical analysis for supplier B under the Deming cost model. The model parameters considered in the analysis are lot size N, cost ratio k_1/k_2, and probability model parameters $(\sigma,\ \tau,\ \gamma)$. A comparison on METC and OSS between the variables sampling model and the corresponding attributes sampling model under the same probability assumptions and cost structure is also presented.

Figure 2 shows the variation of expected total cost with respect to different sample sizes for supplier B under lot size $N = 500$. Our experimental results show that the expected cost as a function of the sample size behaves like the curve shown in Figure 2. This result provides a good reference for us to develop an efficient algorithm for finding the METC and its corresponding OSS.

Define unity cost as the METC per unit; that is, METC$/N$. Figure 3 illustrates the behaviors of the unity cost and the *OSS* as the value of N varies. The unity cost is decreasing and convex in N, which implies that the marginal effect on the cost saving of this model is decreasing as the lot size N becomes larger and larger. On the other hand, the *OSS* is concave and increasing in lot size N. In other words, the marginal increase of the *OSS* slows down as N increases. Figure 4 depicts the effects of the cost ratio k_1/k_2 with fixed $k_2 = \$72.40$ and $N = 500$. The *OSS* decreases approximately linearly, whereas the unity cost increases approximately linearly as the cost ratio increases. It is quite sensible that the increase of k_1 will increase the METC and reduce the size of *OSS*.

Figure 5 presents the effect of the standard deviation σ of the manufacturing process for the items when the process mean τ does not deviate much from the center of specification limits. For supplier B, $\tau = 24.0137$

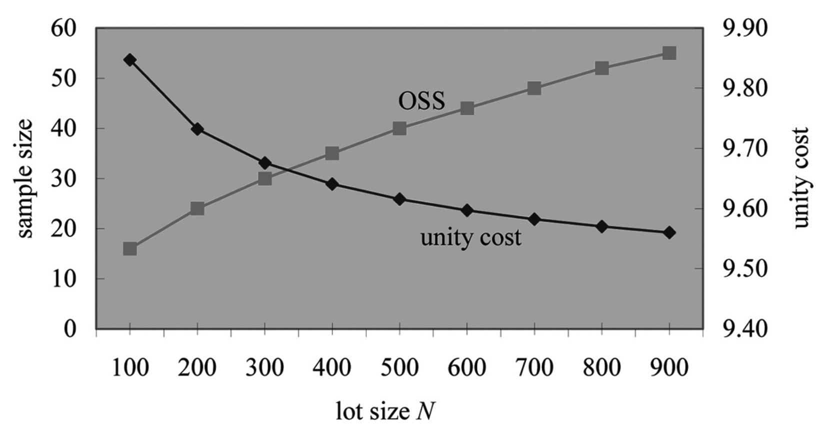

Figure 3. OSS and unity cost versus lot size N.

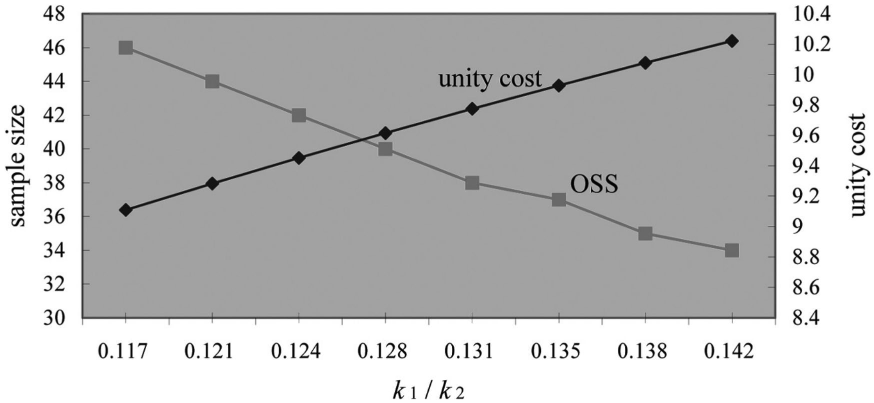

Figure 4. OSS and unity cost versus cost ratio k_1/k_2 with $k_2 = 72.4$.

and $(a + b)/2 = 24$. When σ increases, the probability of an item being conforming becomes smaller and this will make both the OSS and the unity cost increase. A similar situation with the same reasoning happens as the prior mean τ moves further away from the specification mean as shown in Figure 6. The parameter, γ, represents the degree of uncertainty for the unknown mean, U. Figure 7 indicates that the increase of γ reduces the probability of an item being conforming and thus the METC increases, but the OSS is insensitive to the variation of γ.

Table 2 shows that the METC and the OSS for the variables sampling model are smaller than those for the corresponding attributes sampling model under the same probability model and cost structure. This is because in this study the attributes information is derived from the variables information. In such a situation, it is possible to achieve the same power with a sample size in a variable acceptance sampling plan far smaller than the sample size for a derived attributes acceptance sampling plan.

6. CONCLUSION

In this article, the Deming cost model with normal distribution of the performance variables is studied using the Bayesian approach. The prior distribution for the unknown mean is also assumed normally distributed. Numerical integration method is employed to find the optimal solution to the model, which includes the optimal sample size and the acceptance limits for the subsequent action. A numerical result is presented to show how the model parameters, such as lot size, prior distribution, and cost ratio, affect the optimal sample size and the minimum expected total cost. In addition, a method is proposed to formulate the problem as an attributes sampling model under the same probability assumptions and cost structure. This formulation allows us to compute how much cost is saved for using the variable measurement data instead of the corresponding go/no-go data.

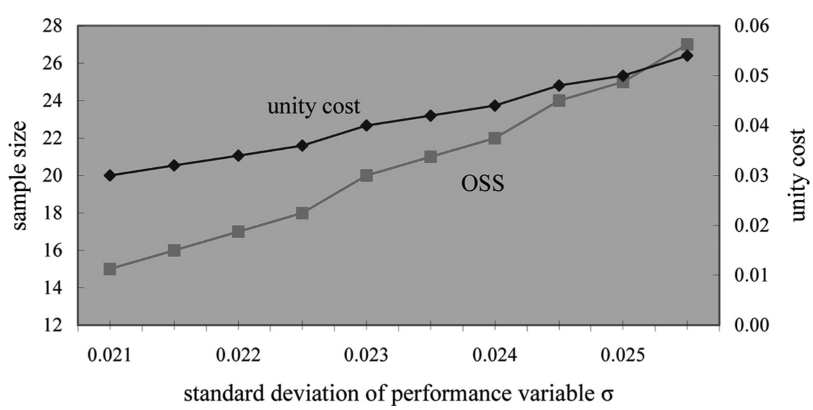

Figure 5. OSS and unity cost versus standard deviation σ.

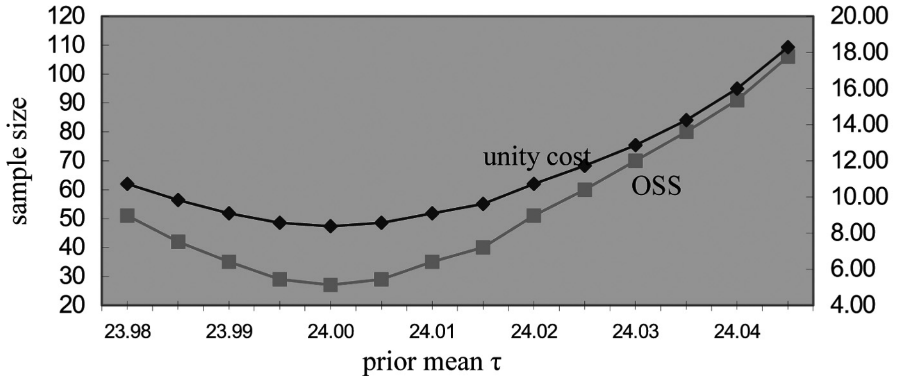

Figure 6. OSS and unity cost versus prior mean τ.

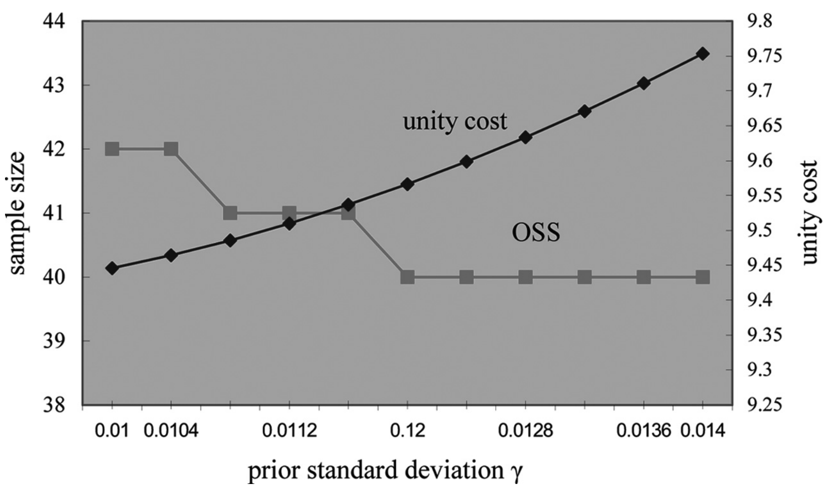

Figure 7. OSS and unity cost versus prior standard deviation γ.

Table 2
Comparison between two sampling models for supplier B

	Variables sampling			Attributes sampling		
Lot size N	OSS	Acceptance limits $[\bar{x}_L, \bar{x}_R]$	Unity cost	OSS	Acceptance number, c	Unity cost
100	16	[23.9754, 24.0161]	9.85	25	4	10.18
200	24	[23.9780, 24.0162]	9.73	46	7	10.02
300	30	[23.9791, 24.0163]	9.68	61	9	9.94
400	35	[23.9797, 24.0164]	9.64	68	10	9.88
500	40	[23.9801, 24.0165]	9.62	83	12	9.84
600	44	[23.9804, 24.0165]	9.60	91	13	9.81
700	48	[23.9807, 24.0165]	9.58	98	14	9.78
800	52	[23.9809, 24.0165]	9.57	106	15	9.76
900	55	[23.9810, 24.0165]	9.56	114	16	9.74

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APPENDIX

The objective function is

$$\underset{0\le n\le N}{\mathrm{Min}} \int_{-\infty}^{\infty} \left[\sum_{y=0}^{n} \{\mathrm{Min}[E(L_{Sn}|n,\bar{x},y),\ E(L_{CN}|n,\bar{x},y)]\} \times \Pr\{y|n,\bar{x}\} \right] \cdot g(\bar{x}|n) d\bar{x}.$$

By taking expectation on Eqs. (1) and (2), we obtain

$$\begin{aligned} E(L_{sn}|n,\bar{x},y) &= n\cdot k_1 + E(M(y)|n,\bar{x},y)\cdot k_1 \\ &\quad + E(Z_{N-n}|n,\bar{x},y)\cdot k_2 \\ &\quad + E(M(Z_{N-n})|n,\bar{x},y)\cdot k_1 \\ E(L_{CN}|n,\bar{x},y) &= n\cdot k_1 + E(M(y)|n,\bar{x},y)\cdot k_1 \\ &\quad + (N-n)\cdot k_1 \\ &\quad + E(M(Z_{N-n})|n,\bar{x},y)\cdot k_1 \end{aligned}$$

We need to compute

(i) $E(Z_{N-n}|n,\bar{x},y)$: The expected number of nonconforming items in the remaining *N-n* items of the lot given the information $(n,\bar{x},y)$.
(ii) $E(M(y)|n,\bar{x},y)$: The expected number of extra inspections to obtain y conforming items given the information $(n,\bar{x},y)$.
(iii) $E(M(Z_{N-n})|n,\bar{x},y)$: The expected number of extra inspections to obtain Z_{N-n} conforming items given the information $(n,\bar{x},y)$.
(iv) $\sum_{y=0}^{n} E(M(y)|n,\bar{x},y)\cdot \Pr\{y|n,\bar{x}\}$.

(i) $E(Z_{N-n}|n,\bar{x},y) = (N-n)(1-E(P(U)|n,\bar{x})$.
For any $U=u$, we have $E(Z_{N-n}|n,\bar{x},y,U=u) = E(Z_{N-n}|n,u) = (N-n)(1-P(u))$.
Therefore,

$$E(Z_{N-n}|n,\bar{x},y,U) = (N-n)(1-P(U))$$

$$\begin{aligned} E(Z_{N-n}|n,\bar{x},y) &= E[E(Z_{N-n}|n,\bar{x},y,U)|n,\bar{x},y] \\ &= (N-n)(1-E(P(U)|n,\bar{x},y)) \\ &= (N-n)(1-E(P(U)|n,\bar{x})). \end{aligned}$$

The posterior of $P(U)$ depends on $(n, \bar{x})$ only.

(ii) $E(M(y)|n, \bar{x}, y) = y \cdot E\big((1/P(U))|n, \bar{x}\big)$

$$E(M(y)|n, \bar{x}, y, U = u) = y \cdot \frac{1}{P(u)}$$

$$\begin{aligned} E(M(y)|n, \bar{x}, y) &= E[E(M(y)|n, \bar{x}, y, U)|n, \bar{x}, y)] \\ &= E\left(y \cdot \frac{1}{P(U)}\middle| n, \bar{x}, y\right) \\ &= y \cdot E\left(\frac{1}{P(U)}\middle| n, \bar{x}, y\right) = y \cdot E\left(\frac{1}{P(U)}\middle| n, \bar{x}\right) \\ &= y \cdot \int_{-\infty}^{\infty} \frac{1}{P(u)} \cdot f(u|n, \bar{x}) du \end{aligned}$$

(iii) $E(M(Z_{N-n})|n, \bar{x}, y) = (N-n)\left[E\left(\frac{1}{P(U)}|n, \bar{x}\right) - 1\right]$

$$\begin{aligned} E(M(Z_{N-n})|n, \bar{x}, y, U = u) &= (N-n)\frac{1-P(u)}{P(u)} \\ &= (N-n)\left(\frac{1}{P(u)} - 1\right) \end{aligned}$$

$$\begin{aligned} &E(M(Z_{N-n})|n, \bar{x}, y) \\ &\quad = E[E(M(Z_{N-n})|n, \bar{x}, y, U)|n, \bar{x}, y) \\ &\quad = (N-n)\left[E\left(\frac{1}{P(U)}|n, \bar{x}\right) - 1\right] \end{aligned}$$

(iv) $\sum_{y=0}^{n} E(M(y)|n, \bar{x}, y) \cdot \Pr\{y|n, \bar{x}\}$

$$\begin{aligned} &= \sum_{y=0}^{n} y \cdot E\left(\frac{1}{P(U)}\middle| n, \bar{x}\right) \cdot \Pr\{y|n, \bar{x}\} \\ &= E\left(\frac{1}{P(U)}\middle| n, \bar{x}\right) \cdot \sum_{y=0}^{n} y \cdot \Pr\{y|n, \bar{x}\} \\ &= E\left(\frac{1}{P(U)}\middle| n, \bar{x}\right) \cdot n \cdot (1 - E(P(U)|n, \bar{x}) \end{aligned}$$

At the second stage decision, decision "*Sn*" is superior to "*CN*" if

$$E(L_{Sn}|n, \bar{x}, y) \le E(L_{CN}|n, \bar{x}, y).$$

$$\begin{aligned} &E(L_{Sn}|n,\bar{x},y) - E(L_{CN}|n,\bar{x},y) \\ &\quad = E(Z_{N-n}|n,\bar{x},y) \cdot k_2 - (N-n) \cdot k_1 \\ &\quad = (N-n) \cdot (1 - E(P(U)|n,\bar{x}) \cdot k_2 - (N-n) \cdot k_1) \\ &\quad \le 0 \end{aligned}$$

Thus, we obtain Eq. (3)

$E(L_{Sn}|n, \bar{x}, y) \le E(L_{CN}|n, \bar{x}, y)$ if and only if

$$1 - E(P(U)|n, \bar{x}) \le \frac{k_1}{k_2}$$

The inner part of the objective function [Eq. (6)] can be simplified as follows:

$$\begin{aligned} &\sum_{y=0}^{n} \{\mathrm{Min}[E(L_{Sn}|n,\bar{x},y), E(L_{CN}|n,\bar{x},y)]\} \cdot \Pr\{y|n,\bar{x}\} \\ &\quad = \sum_{y=0}^{n} \{ \cdot [n \cdot k_1 + E(M(y)|n,\bar{x},y) \\ &\qquad \cdot k_1 + E(M(Z_{N-n})|n,\bar{x},y) \cdot k_1] \\ &\qquad + \mathrm{Min}[E(Z_{N-n}|n,\bar{x},y) \cdot k_2, (N-n) \cdot k_1]\} \cdot \Pr\{y|n,\bar{x}\} \\ &= \sum_{y=0}^{n} \{n \cdot k_1 + y \cdot E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) \cdot k_1 \\ &\quad + (N-n)\left(E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) - 1\right) \cdot k_1 \\ &\qquad + (N-n) \cdot \mathrm{Min}[(1 - E(P(U)|n,\bar{x}) \cdot k_2, k_1]\} \\ &\qquad \cdot \Pr\{y|n,\bar{x}\} = n \cdot k_1 + E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) \\ &\qquad \cdot n \cdot (1 - E(P(U)|n,\bar{x})) \cdot k_1 \\ &\quad + (N-n) \cdot \left[\left(E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) - 1\right) \cdot k_1 \right. \\ &\qquad\qquad \left. + \mathrm{Min}\{(1 - E(P(U)|n,\bar{x})) \cdot k_2, k_1\}\right] \end{aligned}$$

Finally, the objective function becomes

$$\begin{aligned} &= \underset{0 \le n \le N}{\mathrm{Min}} \int_{-\infty}^{\infty} \left\{ n \cdot k_1 + .(N-n) \cdot \left[\left(E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) - 1\right) \right.\right. \\ &\quad \left. \cdot k_1 + \mathrm{Min}\left[\left(1 - E(P(U)|n,\bar{x})\right) \cdot k_2, k_1\right]\right] \\ &\quad \left. + E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) \cdot n \cdot (1 - E(P(U)|n,\bar{x})) \cdot k_1 \right\} \cdot g(\bar{x}|n) d\bar{x} \end{aligned}$$

$$\begin{aligned} &= \underset{0 \le n \le N}{\mathrm{Min}} \left\{ n \cdot k_1 + (N-n) \cdot \left[E\left(\frac{1}{P(U)}\right) - 1\right] \cdot k_1 \right. \\ &\qquad + (N-n). \int_{-\infty}^{\infty} \mathrm{Min}[(1 - E(P(U)|n,\bar{x})) \cdot k_2, k_1] \\ &\qquad \cdot (\bar{x}|n) \cdot d\bar{x} + n \cdot k_1 \cdot \int_{-\infty}^{\infty} E\left(\frac{1}{P(U)}\middle| n,\bar{x}\right) \cdot \\ &\qquad \left. \cdot (1 - E(P(U)|n,\bar{x})) \cdot g(\bar{x}|n) \cdot d\bar{x} \right\} \end{aligned}$$